Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 4 June 2024

Afternoon (Time: 2 hours)

Paper reference **9MA0/01**

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.

$$g(x) = 3x^3 - 20x^2 + (k + 17)x + k$$

where k is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of k.

(3)

If $(x-3)$ is a factor of $g(x)$, $g(3)=0$

$g(3) = 3(3)^3 - 20(3)^2 + (k+17)(3) + k = 0$ 1

$81 - 180 + 3k + 51 + k = 0$

$4k - 48 = 0$ 1

$k = 12$ 1

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Question 1 continued

(Total for Question 1 is 3 marks)

2. (a) Find, in ascending powers of x, the first four terms of the binomial expansion of

$$(1-9x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Give a reason why $x = -\frac{2}{9}$ should **not** be used in the expansion to find an approximation to $\sqrt{3}$

(1)

a)

$$(1-9x)^{1/2} = 1 - \frac{9x}{2} + \frac{(\frac{1}{2})(\frac{1}{2}-1)}{2!}(-9x)^2 + \frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(-9x)^3$$ ① ①

$$= 1 - \frac{9x}{2} - \frac{81x^2}{8} - \frac{729x^3}{16}$$ ①

b) the expansion is only valid for

$|-9x| < 1 \Rightarrow |x| < \frac{1}{9}$

and $x = -\frac{2}{9}$ is outside of this range. ①

Question 2 continued

(Total for Question 2 is 4 marks)

3. $$f(x) = x + \tan\left(\frac{1}{2}x\right) \qquad \pi < x < \frac{3\pi}{2}$$

Given that the equation $f(x) = 0$ has a single root α

(a) show that α lies in the interval [3.6, 3.7]

(2)

(b) Find $f'(x)$

(2)

(c) Using 3.7 as a first approximation for α, apply the Newton–Raphson method once to obtain a second approximation for α. Give your answer to 3 decimal places.

(2)

a) $f(3.6) = 3.6 + \tan\left(\frac{3.6}{2}\right) = -0.686\ldots < 0$

$f(3.7) = 3.7 + \tan\left(\frac{3.7}{2}\right) = 0.211\ldots > 0$ 1

- since there is a change in sign
- and $f(x)$ is continuous in this interval
- there is a root α in [3.6, 3.7] 1

b) $f(x) = x + \tan\left(\frac{1}{2}x\right)$ 1

$f'(x) = 1 + \frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ 1

c) $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 3.7 - \frac{3.7 + \tan\left(\frac{3.7}{2}\right)}{1 + \frac{1}{2}\sec^2\left(\frac{3.7}{2}\right)}$$ 1

$= 3.67205\ldots$

$= 3.672$ (3dp) 1

Question 3 continued

(Total for Question 3 is 6 marks)

4. Given that $y = x^2$, use differentiation from first principles to show that $\dfrac{dy}{dx} = 2x$

(3)

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \quad (1)$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} \quad (1)$$

$$= \lim_{h \to 0} (2x + h)$$

$$= 2x \quad (1)$$

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Question 4 continued

(Total for Question 4 is 3 marks)

5. The function f is defined by

$$f(x) = \frac{2x - 3}{x^2 + 4} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2 + bx + c}{(x^2 + 4)^2}$$

where a, b and c are constants to be found.

(3)

(b) Hence, using algebra, find the values of x for which f is decreasing. You must show each step in your working.

(3)

a) $f(x) = \dfrac{2x-3}{x^2+4}$

1 1

$f'(x) = \dfrac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2} = \dfrac{2x^2+8-4x^2+6x}{(x^2+4)^2}$

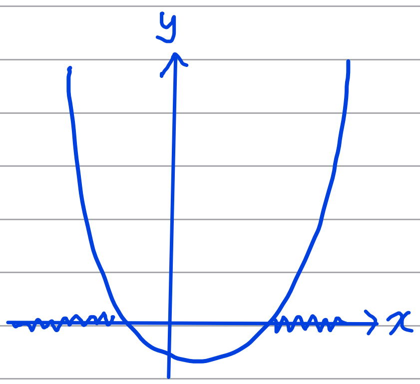
$= \dfrac{-2x^2+6x+8}{(x^2+4)^2}$ 1

b) $(x^2+4)^2 > 0 \ \forall x \ \therefore$ f is decreasing when

$-2x^2+6x+8 < 0$

$x^2 - 3x - 4 > 0$

$(x+1)(x-4) > 0$



critical values: $x = 4, x = -1$ 1

choose outside region:

$x < -1$ or $x > 4$

1 1

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Question 5 continued

(Total for Question 5 is 6 marks)

6.

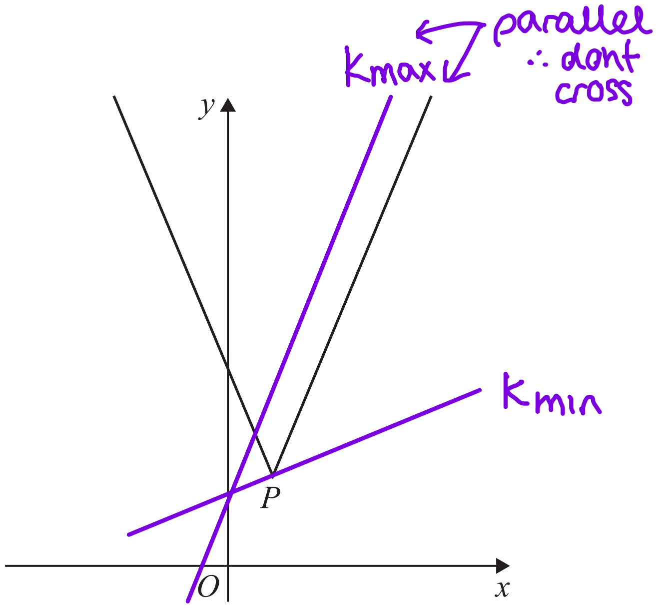

Figure 1

Figure 1 shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point P, shown in Figure 1.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$16 - 4x = 3|x - 2| + 5$$

(2)

A line l has equation $y = kx + 4$ where k is a constant.

Given that l intersects $y = 3|x - 2| + 5$ at 2 distinct points,

(c) find the range of values of k.

(2)

a) P has $|x-2| = 0$ $\quad y = 3|2-2| + 5$

$x - 2 = 0 \quad = 5$

$x = 2$ 1

$P(2, 5)$ 1

P 7 5 6 9 3 A 0 1 2 4 4

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Question 6 continued

b) $16 - 4x = 3|x-2| + 5$ consider solutions for $|x-2| > 0$ since there are no solutions for $|x-2| < 0$

$16 - 4x = 3(x-2) + 5$ (1)

$16 - 4x = 3x - 6 + 5$

$17 = 7x$

$x = \frac{17}{7}$ (1)

c) if $k \geq 3$, there is only one solution as $y = kx + 4$ will never cross the positive branch of $y = 3|x-2| + 5$

if $y = kx + 4$ touches P there is only one solution.

sub in $x=2, y=5$: $5 = 2k + 4$

$\Rightarrow k = \frac{5-4}{2} = \frac{1}{2}$ (1)

so gradient must be steeper than $\frac{1}{2}$

$\therefore \frac{1}{2} < k < 3$ (1)

(Total for Question 6 is 6 marks)

7.

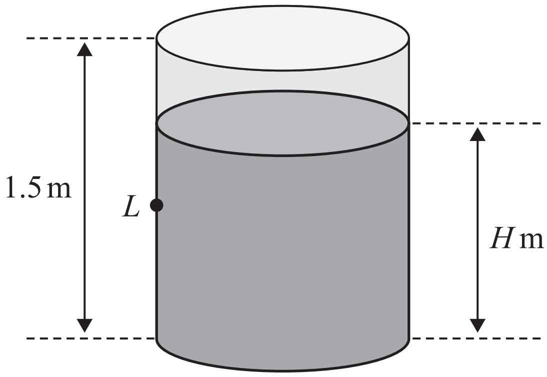

Figure 2

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water. $\Rightarrow$ when $t=0$, $H=1.5$

The water starts to leak from a small hole, at a point L, in the side of the tank.

While the tank is leaking, the depth, H metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where t hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where A and B are constants to be found,

(3)

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

(3)

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

(2)

a) $\frac{dH}{dt} = -0.12e^{-0.2t}$

$H = \frac{-0.12}{-0.2}e^{-0.2t} + C$

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Question 7 continued

$H = 0.6e^{-0.2t} + c$ 1

sub in $t=0$, $H=1.5$

$1.5 = 0.6e^{0} + c$

$c = 0.9$ 1

$H = 0.6e^{-0.2t} + 0.9$ 1

b) sub in $H = 1.2$

$1.2 = 0.6e^{-0.2t} + 0.9$

$0.6e^{-0.2t} = 0.3$ 1

$e^{-0.2t} = 0.5$

$-0.2t = \ln 0.5$ 1

$t = -5\ln 0.5 = 3.47$ hours

$= 3h + (0.47 \times 60)$ minutes

$= 3$ hours 28 minutes 1

c) as t gets large, $e^{-0.2t} \to 0$ so $H \to 0 + 0.9$ 1

so the hole is 0.9m from the bottom

1

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

(a) Find fg(2)

(2)

(b) Find g^{-1}

(3)

(c) (i) Find gf(x), giving your answer as a simplified fraction.

(ii) Deduce the range of gf(x).

(3)

The function h is defined by

$$h(x) = 2x^2 - 6x + k \qquad x \in \mathbb{R}$$

where k is a constant.

(d) Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.

(3)

a) $g(2) = \frac{5}{2(2)-9} = -1$ ① $\quad f(-1) = 4 - 3(-1)^2$
$= 1$

$fg(2) = 1$ ①

b) $y = \frac{5}{2x-9} \Rightarrow 2xy - 9y = 5$

$2xy = 5 + 9y$ ① Rearrange for x

$x = \frac{5+9y}{2y}$ ①

$g^{-1}(x) = \frac{5+9x}{2x}$, $x \neq 0$, $x \in \mathbb{R}$ ①

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Question 8 continued

c) (i) $gf(x) = \frac{5}{2(4-3x^2)-9} = \frac{5}{-6x^2-1} = -\frac{5}{1+6x^2}$ ① ①

(ii) $1+6x^2 > 0 \; \forall x$, so $gf(x) < 0$ $\forall$ means "for all"

$\min -\frac{5}{1+6x^2} = \max \frac{5}{1+6x^2}$.

this happens when the denominator is as small as possible (i.e. $x=0$)

$\Rightarrow \max \frac{5}{1+6x^2} = 5$, $\min -\frac{5}{1+6x^2} = -5$

$\therefore \quad -5 \leq gf(x) < 0$ ①

d) $f(x) = h(x)$

$4 - 3x^2 = 2x^2 - 6x + k$

$5x^2 - 6x + k - 4 = 0$ ①

no real solutions so "$b^2 - 4ac$" < 0

$6^2 - 4(5)(k-4) < 0$ ①

$36 - 20k + 80 < 0$

$20k > 116$

$k > 5.8$ ①

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9. The first 3 terms of a geometric sequence are

$$3^{4k-5} \qquad 9^{7-2k} \qquad 3^{2(k-1)}$$

where k is a constant.

(a) Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$ **(3)**

(b) Hence find the sum to infinity of the geometric sequence. **(3)**

a) $\frac{9^{7-2k}}{3^{4k-5}} = \frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2(k-1)}}{3^{2(7-2k)}}$ 1

$3^{4(7-2k)} = 3^{4k-5} \times 3^{2k-2}$ 1

$28 - 8k = 4k - 5 + 2k - 2$

$14k = 35$

$k = \frac{5}{2}$ 1

b) $a = 3^{4(\frac{5}{2})-5} = 243$

$r = \frac{9^{7-2(\frac{5}{2})}}{3^{4(\frac{5}{2})-5}} = \frac{1}{3}$ 1

1

$S_\infty = \frac{a}{1-r} = \frac{243}{1-\frac{1}{3}} = \frac{729}{2}$ 1

Question 9 continued

(Total for Question 9 is 6 marks)

10.

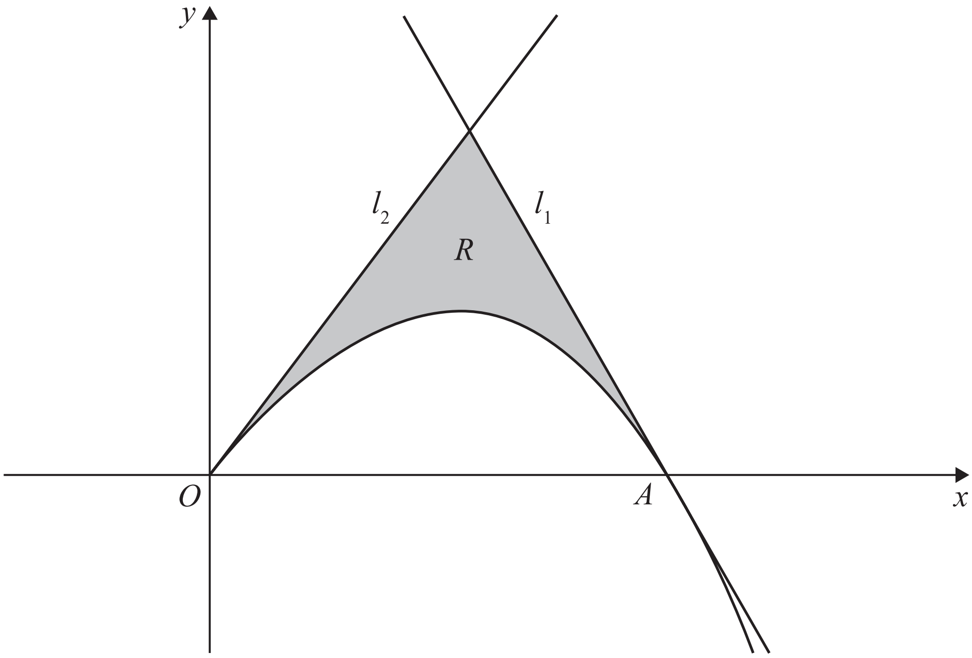


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the x-axis at the point A.

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A.

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R, shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R.

(5)

a) when $x=4$, $y=8(4)-(4)^{5/2} = 32-32=0$ ✓ 1

P 7 5 6 9 3 A 0 2 4 4 4

Question 10 continued

b) l_1 = tangent to curve at A

$y = 8x - x^{5/2}$

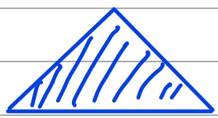
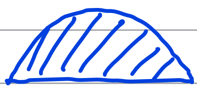
$\frac{dy}{dx} = 8 - \frac{5}{2}x^{3/2}$ (1)

$\left.\frac{dy}{dx}\right|_{x=4} = 8 - \frac{5}{2}(4)^{3/2} = -12$ (1)

$\therefore\ y - 0 = -12(x-4)$

$y = -12x + 48$ (1)

$12x + y = 48$

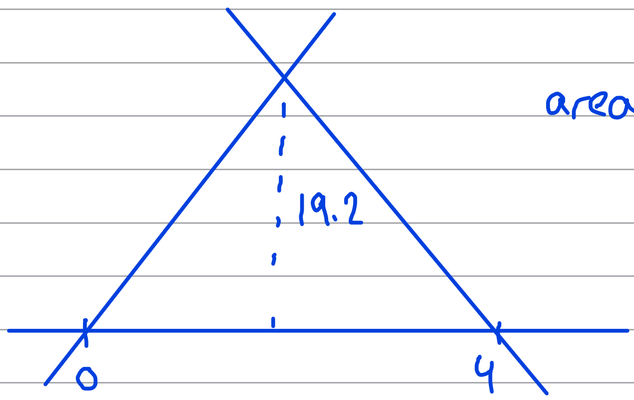
c) R =  − 

coordinates of intersection of l_1 and l_2:

$12x + (8x) = 48$

$x = \frac{48}{20} = 2.4$

$y = 8(2.4) = 19.2$ (1)



area of triangle $= \frac{1}{2} \times 19.2 \times 4$

$= 38.4$ (1)

Question 10 continued

area under curve $= \int_0^4 (8x - x^{5/2})\,dx$

1

$= \left[4x^2 - \frac{2}{7}x^{7/2}\right]_0^4 = 4(4)^2 - \frac{2}{7}(4)^{7/2} = \frac{192}{7}$

1 1

$R = 38.4 - \frac{192}{7} = \frac{384}{35}$

Question 10 continued

(Total for Question 10 is 9 marks)

11.

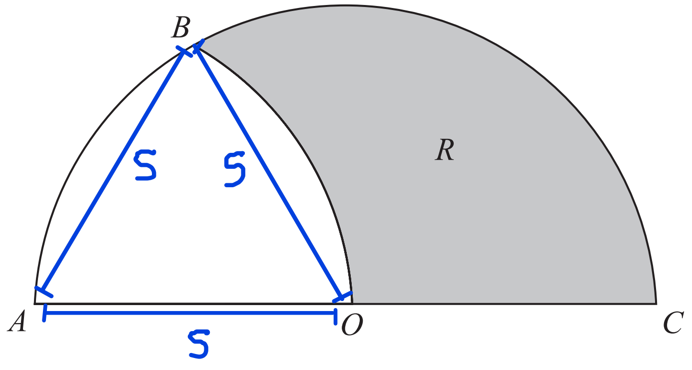


Figure 4

Figure 4 shows the design of a badge.

The shape $ABCOA$ is a semicircle with centre O and diameter 10 cm.

OB is the arc of a circle with centre A and radius 5 cm.

The region R, shown shaded in Figure 4, is bounded by the arc OB, the arc BC and the line OC.

Find the exact area of R.

Give your answer in the form $(a\sqrt{3} + b\pi)\,\text{cm}^2$, where a and b are rational numbers.

(4)

$\triangle ABO$ is equilateral $\Rightarrow \angle BAO = \frac{\pi}{3}$ 1

segment AB = sector ABO − triangle ABO

$= \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(5)(5)\sin\frac{\pi}{3}$ 1

$\frac{1}{2}r^2\theta$ $\qquad$ $\frac{1}{2}ab\sin C$

$= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

ABO = triangle ABO + 2 × segment AB

$= \frac{1}{2}(5)(5)\sin\frac{\pi}{3} + 2\left(\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}\right)$

$= \frac{25\pi}{3} - \frac{25\sqrt{3}}{4}$

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Question 11 continued

$R = \text{semicircle} - ABO$

$= \frac{1}{2}\pi(5)^2 - \left(\frac{25\pi}{3} - \frac{25\sqrt{3}}{4}\right)$ 1

$= \frac{25\sqrt{3}}{4} + \frac{25\pi}{6}$ 1

(Total for Question 11 is 4 marks)

12. (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of K and give the value of α, in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R, is modelled by the equation

$$R = A + 140\cos(30t)° - 480\sin(30t)°$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)°$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

a) $140\cos\theta - 480\sin\theta = K\cos(\theta + \alpha)$

$= K\cos\theta\cos\alpha - K\sin\theta\sin\alpha$

$K\cos\alpha = 140$ $\Rightarrow K = \sqrt{140^2 + 480^2} = 500$ (1)

$K\sin\alpha = 480$

$\tan\alpha = \dfrac{480}{140} \Rightarrow \alpha = 73.74°$ (2dp) (1)

$\therefore 500\cos(\theta + 73.74)°$ (1)

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Question 12 continued

b)(i) $R = A + 500\cos(30t + 73.74)^{\circ}$

max $R = 1500$, when $\cos(30t + 73.74)^{\circ} = 1$

$1500 = A + 500$
$A = 1000$

$R = 1000 + 500\cos(30t + 73.74)^{\circ}$ (1)

(ii) R_{min} is when $\cos(30t + 73.74) = -1$

$R = 1000 - 500$
$= 500$ (1)

c) In the middle of April, $t = 3.5$

$R = 1000 + 500\cos(30(3.5) + 73.74)$ (1)

$= 500.1\ldots$

which is close to 500 so the model is suitable. (1)

d) F_{min} is when $\sin(30t + 70) = -1$

$\Rightarrow 30t + 70 = 270$ (1)

$t = \frac{20}{3}$ (1)

sub $t = \frac{20}{3}$ into R:

$R = 1000 + 500\cos\left(30\left(\frac{20}{3}\right) + 73.74\right)$ (1)

$= 1032.6\ldots$
$= 1033$ rabbits (1)

Question 12 continued

Question 12 continued

(Total for Question 12 is 11 marks)

13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta$$

(4)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x = k\pi a^2$$

where k is a constant to be found.

(4)

a) $x = a\sin^2\theta$

$\frac{dx}{d\theta} = 2a\sin\theta\cos\theta$ ①

$dx = 2a\sin\theta\cos\theta\, d\theta$

limits: when $x=0$, $\theta = 0$
when $x=a$, $\theta = \pi/2$ ①

consider $\sqrt{a-x}$

$\sqrt{a-x} = \sqrt{a-a\sin^2\theta} = \sqrt{a(1+\sin^2\theta)} = \sqrt{a}\cos\theta$ ①

$\int_0^a x^{1/2}\sqrt{a-x}\,dx = \int_0^{\pi/2}\sqrt{a}\sin\theta(\sqrt{a}\cos\theta)(2a\sin\theta\cos\theta)\,d\theta$

$= 2a^2\int_0^{\pi/2}\sin^2\theta\cos^2\theta\,d\theta = 2a^2\int_0^{\pi/2}\left(\frac{1}{2}\sin 2\theta\right)^2 d\theta$

$= \frac{1}{2}a^2\int_0^{\pi/2}\sin^2 2\theta\,d\theta$ ①

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Question 13 continued

b) $\frac{1}{2}a^2\int_0^{\pi/2} \sin^2 2\theta \, d\theta$

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$\Rightarrow \sin^2 2\theta = \frac{1-\cos 4\theta}{2}$

$= \frac{1}{2}a^2 \int_0^{\pi/2} \frac{1-\cos 4\theta}{2} \, d\theta$ (1)

$= \frac{1}{4}a^2 \int_0^{\pi/2} (1 - \cos 4\theta) \, d\theta$

$= \frac{1}{4}a^2 \left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\pi/2}$ (1) (1)

$= \frac{1}{4}a^2 \left[\left(\frac{\pi}{2} - \frac{1}{4}\sin 2\pi\right) - \left(0 - \frac{1}{4}\sin 0\right)\right]$

$= \frac{1}{8}\pi a^2$ (1)

Question 13 continued

Question 13 continued

(Total for Question 13 is 8 marks)

14. A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time t seconds, the radius of the balloon is r cm.

(a) Write down a differential equation to model this situation.

(1)

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of 0.9 cm s^{-1}

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

(5)

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.

(2)

(d) Suggest a limitation of the model.

(1)

a) $\frac{dr}{dt} = \frac{k}{\sqrt{r}}$ 1

b) when $t=10$, $r=16$, $\frac{dr}{dt} = 0.9$

$0.9 = \frac{k}{\sqrt{16}} \Rightarrow k = 3.6$ 1

$\frac{dr}{dt} = \frac{3.6}{\sqrt{r}} \Rightarrow \int \sqrt{r}\, dr = \int 3.6\, dt$ 1

$\frac{2}{3} r^{3/2} = 3.6t + c$ 1

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Question 14 continued

sub $t=10$, $r=16$

$$\frac{2}{3}(16)^{3/2} = 3.6(10)+c$$

$$c = \frac{20}{3} \quad 1$$

$$\frac{2}{3}r^{3/2} = 3.6t + \frac{20}{3}$$

$$\Rightarrow \quad r^{3/2} = 5.4t+10 \quad 1$$

c) sub in $t=20$

$$r^{3/2} = 5.4(20)+10$$
$$= 118$$

$$r = (118)^{2/3} = 24.057\ldots$$
$$= 24.1\text{ cm (nearest mm)} \quad 1$$

d) The model may not be suitable indefinitely, as the balloon may burst. 1

Question 14 continued

Question 14 continued

(Total for Question 14 is 9 marks)

15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k.

(2)

(ii) A student was asked to prove by contradiction that

"There are no positive integers x and y such that $(3x + 2y)(2x - 5y) = 28$"

The start of the student's proof is shown below.

Assume that positive integers x and y exist such that
$(3x + 2y)(2x - 5y) = 28$

If $3x + 2y = 14$ and $2x - 5y = 2$

$$\left.\begin{aligned} 3x + 2y &= 14 \\ 2x - 5y &= 2 \end{aligned}\right\} \Rightarrow x = \frac{74}{19},\ y = \frac{22}{19} \quad \text{Not integers}$$

Show the calculations and statements needed to complete the proof.

(4)

(i) $k^2 - 4k + 5 = (k-2)^2 - 4 + 5 = (k-2)^2 + 1$ 1

since $(k-2)^2 \geq 0 \ \forall k$,

$k^2 - 4k + 5$ is always positive. 1

(ii) if x and y are integers, $(3x+2y)$ and $(2x-5y)$ are also integers.

Possible combinations of integers to make product 28:

$(14, 2)$ ✓ $(2, 14)$ $(7, 4)$ $(4, 7)$ $(28, 1)$ $(1, 28)$

$(-14, -2)$ $(-2, -14)$ $(-7, -4)$ $(-4, -7)$ $(-28, -1)$ $(-1, -28)$

but since both x and y are positive, $3x+2y$ is positive. so we only need to consider the top row.

Question 15 continued

$(2,14)$: $3x+2y=2$, $2x-5y=14$ $\Rightarrow x=2,\ y=-2$ ✗ $(y<0)$ (1)

$(7,4)$: $3x+2y=7$, $2x-5y=4$ $\Rightarrow x=\frac{43}{19},\ y=\frac{2}{19}$ ✗ not integers (1)

$(4,7)$: $3x+2y=4$, $2x-5y=7$ $\Rightarrow x=\frac{34}{19},\ y=-\frac{13}{19}$ ✗ not integers

$(28,1)$: $3x+2y=28$, $2x-5y=1$ $\Rightarrow x=\frac{142}{19},\ y=\frac{53}{19}$ ✗ not integers

$(1,28)$: $3x+2y=1$, $2x-5y=28$ $\Rightarrow x=\frac{61}{19},\ y=-\frac{82}{19}$ ✗ not integers (1)

all cases have been considered, and in each case x and y are not positive integers.

Hence proven by contradiction. (1)

Question 15 continued

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS